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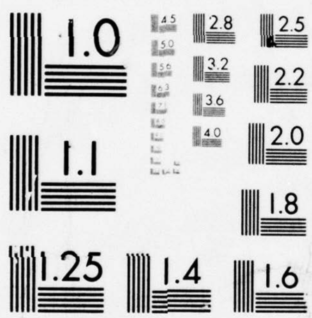

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NAVAL SURFACE WEAPONS CENTER REDUCTION AND ANALYSIS OF DOPPLER SATELLITE RECEIVERS USING THE CELEST COMPUTER PROGRAM

by

JAMES W. O'TOOLE
Warfare Analysis Department

MARCH 1977

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The CELEST Computer Program uses raw Doppler data to determine satellite orbits. It provides diagnostic information on the quality of the orbits and the data used in producing those orbits. The basic technique employed is one of weighted least squares where the data is edited and weighted within the program. An iterative capability exists for nonlinear problems. Trajectories are formed by directly integrating the equations of motion in an inertial frame. The force equation has components due to earth, sun, and

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moon gravity, solar radiation, thrust, atmospheric drag, solar and lunar tidal distortion. A satellite frequency offset error can be determined, and the program has the facility for determining unknown receiver locations. The computer program occupies 130 K octal units of memory, is structured as nine major overlays, and is completely written in Fortran. The program is primarily operated on a sixty bit CDC 6700 computer.

FOREWORD

This report gives an overview of the orbit computation program CELEST used by the Naval Surface Weapons Center in processing Doppler data to position both satellites and surface based Doppler receivers. The Doppler system has been in use since 1962 by the U.S. Navy and has been used at the Naval Surface Weapons Center, Dahlgren, Virginia primarily for Defense Department related geodetic studies.

This CELEST computer program uses raw Doppler data to determine satellite orbits. It provides diagnostic information on the quality of the orbits. The basic technique employed is one of weighted least squares where the data is edited and weighted within the program. An iterative capability exists for nonlinear problems. Trajectories are formed by directly integrating the equations of motion in an inertial frame. The force equation has components due to Earth, Sun and Moon gravity, solar radiation, thrust, atmospheric drag, solar and lunar tidal distortion. A satellite frequency offset error can be determined and the program has the facility for determining unknown receiver locations. The computer program occupies 130K octal units of memory, is structured as nine major overlays and is completely written in fortran. The program is primarily operated on a sixty bit CDC 6700 computer at Dahlgren, two IBM 1108 computers at the Defense Mapping Agency Aerospace Center and Topographic Center, on a CDC 6700 at Cambridge Research Laboratory and an SEL 86 located at Naval Space Surveillance Center, Dahlgren, Virginia.

This report was prepared by James W. O'Toole and reviewed by Richard J. Anderle of the Astronautics and Geodesy Division.

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MEASUREMENTS

The majority of Doppler measurements today are taken by Geoceiver or Geoceiver type equipment. This equipment integrates the Doppler effect on the transmitted frequency f_s over approximately 30 seconds to obtain a measurement equivalent to range difference. Letting f_r be the receiver generated frequency, slightly offset from f_s, and ρ indicate range from the receiver to the satellite we have

(1) $$\text{Doppler} = \dot{N}_c = f_r - f_s(1 - \dot{\rho}/c)$$

Integrating fives

(2) $$\rho_2 - \rho_1 = {}^{c}/f_s\,[N_c - (f_r - f_s)\,(t_2 - t_1)]$$

where $t_2 - t_1 \cong 30$ sec.

c = velocity of light

$$\rho = \left|\underset{\text{satellite}}{r(\text{emission time})} - \underset{\text{receiver}}{r(\text{reception time})}\right|$$

Older Doppler equipment which integrates over a time period less than a second treats the data as instantaneous range rate. This is done by solving equation (1) for $\dot{\rho}$ and assigning the midpoint of the integration interval as the time of observation.

STRUCTURAL OVERVIEW

The basic program modules are indicated in Diagram 1. Coordinates of the sun and moon are retained at one day and one half day intervals respectively on the Sun-Moon file. They are in the Mean Inertial System of 1950.0 and ephemeris time. In addition values for the inertial to earth fixed coordinate transformation are retained on this file. These values are the nutation in longitude and obliquity of the ecliptic of the sun, Besselian day numbers and the equation of the equinoxes. The Satellite Table contains offset frequency values to the broadcast frequency of individual satellites. The Gravity file contains spherical harmonic coefficients.

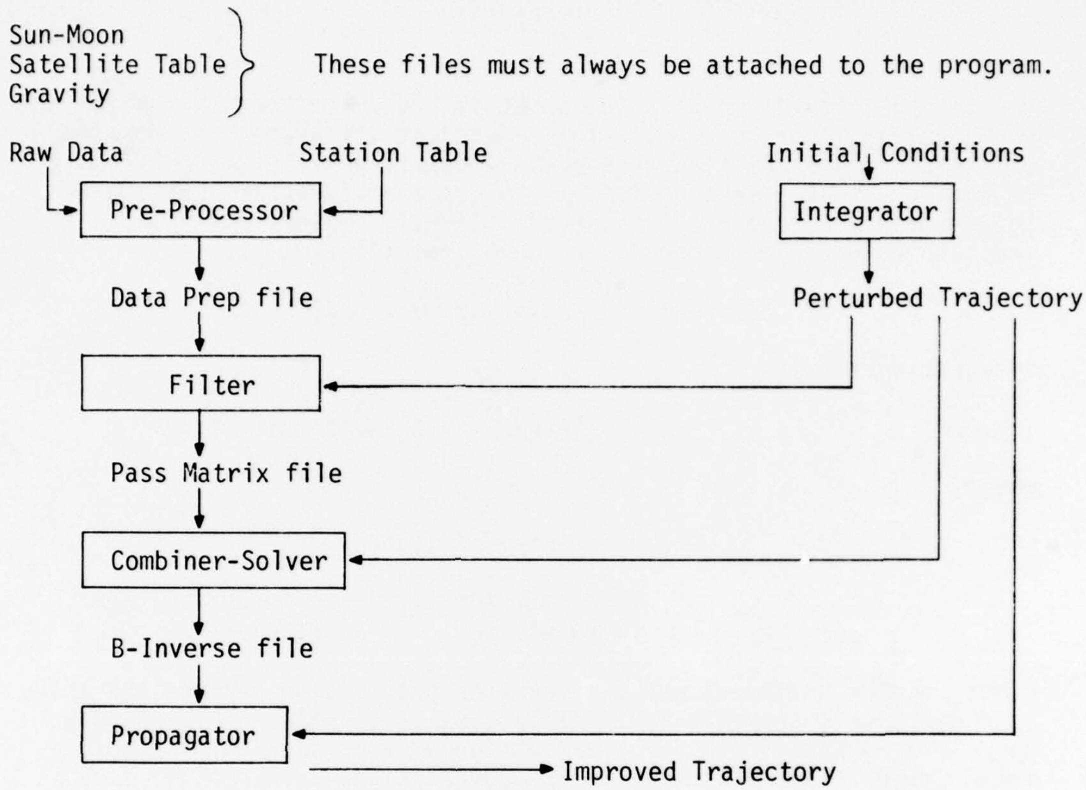


Diagram 1

Basic Program Modules

The program utilizes various pre-processors as each one is designed to process a specific type of data. Pre-processors perform the function of preliminary data editing, time correcting and placing the data in a specific format on the Data Prep file.

Final data point editing and the assignment of weights take place in the filter module. This function is performed by fitting a satellite trajectory to one pass of data at a time. Points are removed which have a residual, from the fitted orbit, greater than twice the computed standard deviation. This procedure is iterated until no further outlying points are identified. At this point the RMS of residuals, from the fitted orbit, is assigned as the standard deviation of an observation. The reciprocal of this value is assigned as a weight. Least square normal equations are made up using the untagged points and their weights.

The normal equations contain a subset of six osculating orbital elements, one drag scaling constant, three thrust parameters, a radiation parameter, three receiver station coordinate parameters, a refraction correction parameter, frequency bias and frequency drift parameters. The subset is determined by the nature of the problem being solved. The normal equation, the time of closest approach and the RWS of residuals are written to a file (Pass Matrix file).

Ordinarily the Pass Matrix file would be the only thing required to complete processing as the remaining task is that of summing matrices and performing an inversion. The solution area is designed however, to determine solutions over various time domains with an option for drag segmentation. This process requires access to partial derivatives from the trajectory. Segmentation is a process whereby the effect of perturbing x number of drag segments independently is desired as part of the solution. The program accomplishes this by storing the effect of perturbing any number of drag segments an equal amount in the pass normal matrix via

$$\frac{\partial \text{ Data}}{\partial \text{ Drag Parameter}} = \frac{\partial D}{\partial C_D}$$

and using the trajectory partials in the solution area to transform the pass matrix value

$$\frac{\partial D}{\partial C_D}$$

into its associated values $\frac{\partial D}{\partial C_{D_i}}$. Each $\frac{\partial D}{\partial C_{D_i}}$ represents the

perturbation in data due to independently perturbing the drag coefficient in the ith segment of the drag force. This permits a flexible treatment of drag and is especially designed for short arc processing.

The final step in the orbit determination process is generating a refined ephemeris. This is done by using trajectory partials to compute the effect on the reference orbit due to parameter changes indicated in the solution. Trajectory values beyond the reference trajectory time span must of course be generated via integration.

The central theme on which both the Celest program and an independent Celest Station Position program operates is the data bank concept of Diagram 2.

It is pass normal matrices which are saved in the data bank. The matrices become the data for most work although raw data is saved for more fundamental studies. By a continued use of matrices, trajectories and the principle of first order matrix adjustment, due to a change in the reference parameter values, orbit refinement and station positioning can be carried on with a minimum of data reprocessing.

DATA FILTERING

Point filtering proceeds with the philosophy that model error during a single pass can be removed by adjusting the orbit in the along track and radial directions together with removing frequency and tropospheric refraction error. The ionospheric error has been removed at the receiving station by gathering dual frequency data and eliminating the first order ionospheric effect.

The filter process is implemented by forming a pass normal matrix on a reference orbit.

$$\begin{bmatrix} B_{\text{orbit, orbit}} & B_{\text{orbit, bias}} \\ & \\ * & B_{\text{bias, bias}} \end{bmatrix} \begin{bmatrix} \Delta\text{orbit} \\ \\ \Delta\text{bias} \end{bmatrix} = \begin{bmatrix} E_{\text{orbit}} \\ \\ E_{\text{bias}} \end{bmatrix}$$

The orbit section always contains six osculating orbital elements at some epoch time. The bias section contains receiver station coordinates, a frequency bias parameter which measures the deviation of the satellite frequency offset from an assumed value and a refraction correction giving the percent deviation from the Hopfield tropospheric model. A transformation ϕ is formed which takes normal

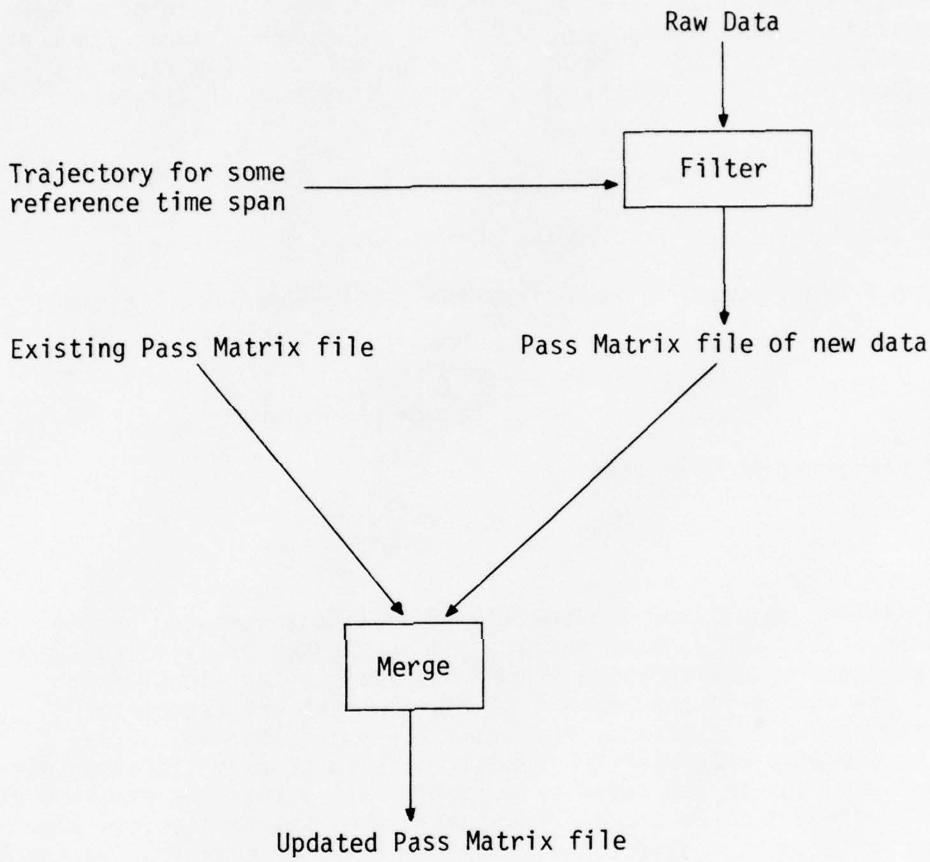


Diagram 2

Pass Matrix Data Bank Concept

equations presented in osculating orbital elements and reforms them in position velocity components resolved in a special local frame at TCA. The local frame is defined by the station-satellite range (ρ) and along track ($\dot{\rho}$) vectors at the satellites time of closest approach.

$$R = [\hat{\rho},\ \hat{\dot{\rho}},\ \widehat{\rho x \dot{\rho}}]$$

where the ^ indicates unit vectors.

The epoch transformation going from the osculating orbital element time to TCA is

$$\psi(\mathrm{TCA}) = \frac{\partial(x,\ \dot{x})\ (\mathrm{TCA})}{\partial e_o\ (\text{epoch elements})}$$

The transformation ϕ is then

$$\phi = \bar{R}\psi \qquad \bar{R} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$$

A solution is determined and the RWS of residuals computed on the adjusted orbit. Data point weights are recomputed by dividing their present value by the adjusted RWS of residuals. Individual data points are tagged if the product of their weight and associated adjusted residual is greater than two (2 sigma filtering). The process iterates reforming the normal equation at each iteration with the untagged points and improved weights. All points are examined at each iteration and the process terminates when the same points are tagged on successive iterations. The final normal equation and the RWS of unadjusted residuals using untagged points is stored on the Pass Matrix file. Basic technical procedures used in this work are given below.

STATION COORDINATE CONVERSION

3) $$r_{s_o} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} = (A + h_s) \begin{pmatrix} \cos Q_s & \cos \lambda_o \\ \cos Q_s & \sin \lambda_o \\ \sin Q_s & \end{pmatrix} - Ae^2 \begin{pmatrix} 0 \\ 0 \\ \sin Q_s \end{pmatrix}$$

where

r_{s_o} = earth fixed station coordinates

h_s = Station geodetic height above a reference ellipsoid

e = eccentricity of the reference ellipsoid

$$= [(2 - f)f]^{\frac{1}{2}} = \left[\frac{2 - \frac{1}{EL}}{EL}\right]^{\frac{1}{2}}$$

f = flattening

EL = oblateness

= 298.25

a_e = semi-major axis of the reference ellipsoid

= 6378.145 km

Q_s = station geodetic latitude

λ_o = longitude from Greenwich meridian

$A = {}^{a_e}/(1 - e^2 \sin^2 Q_s)^{\frac{1}{2}}$

RANGE AND RANGE RATE COMPUTATION

$$\text{(4)} \qquad \text{Range} = \rho(t_r) = r(t_e) - r_s(t_r)$$

where $t_e = t_r - (\rho * \rho)^{\frac{1}{2}}/c$ t_r = reception time and t_e is determined by iteration starting with $t_e = t_r$.

$$\text{(5)} \qquad \dot{\rho} = \dot{r}(t_e) - \dot{r}_s(t_r) - \frac{\rho * [\dot{r}(t_e) - \dot{r}_s(t_r)]\ \dot{r}(t_e)}{c(\rho * \rho)^{\frac{1}{2}}}$$

where $\dot{r}_s = \bar{\omega}\ x r_s$ $\qquad$ r_s = inertial station location

$\ddot{r}_s = \bar{\omega}\ x \dot{r}_s$ $\qquad$ $= (ABCD)^* \, r_{s_o}$

$\bar{\omega} = (CD)^* \begin{pmatrix} 0 \\ 0 \\ \tilde{\omega} \end{pmatrix}$

CD = Precession Nutation transformation

$\tilde{\omega}$ = Earths mean sideral rotation rate determined by the time lapse between successive transits of the mean equinox.

= .7292115855 E(-04) radians/sec.

TIME OF CLOSEST APPROACH (TCA) COMPUTATION

Define

$$t_{i+1} = t_i - (\rho * \dot{\rho})_{t_i} / (\dot{\rho} * \dot{\rho} + \rho * \ddot{\rho}) t_i$$

Set t_1 = value from the observation file and iterate until $|t_{i+1} - t_i| < .05$ sec. or maximum iteration count is reached.

COMPUTATION OF ZENITH ANGLE

(6) $$Z_V = \tan^{-1} ([1-(\hat{\rho} * \hat{U}_S)^2] / \hat{\rho} * \hat{U}_S)$$

$$U_S = (ABCD) * \begin{pmatrix} s_1 \\ s_2 \\ s_3/(1-e^2) \end{pmatrix}$$ s_i computed from (3) with $h_s = 0$

REFRACTION MODEL (HOPFIELD)

N_{T1} = dry term

N_{T2} = wet term

$R_0 = |r_{so}|$

$R_1 = R_0 + 40.1 + .149T$

$R_2 = R_0 + 12.0$

$N_1 = [(.776)\ E\text{-}04]^P / T_k$

$N_2 = [(.373)\ E\text{-}02]^E / T_k^2$

E = water vapor pressure

H = humidity (percent)

P = pressure (millibars)

T_k = Temperature (Kelvin)

= T + 273 (deg centigrade)

H = 80%

$P = 1013.25\ \text{Exp}(-.119913 h_s)$

$T = 15 - 6.5 h_s$

$E \quad = H \exp(-37.2465 + .213166T_k - .000256908T_k^2)$

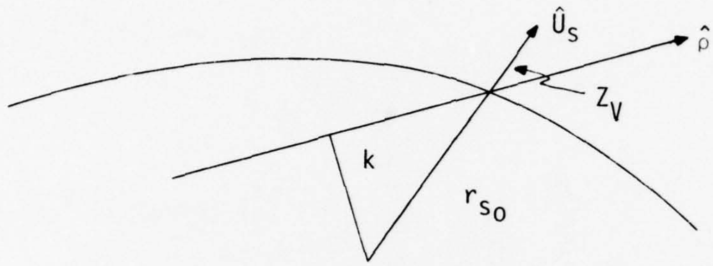
$n \quad =$ index of refraction

$n-1 = N_{T_1} + N_{T_2}$

$N_{T_i} = N_i \left[\dfrac{r^2 - R_i^2}{R_o^2 - R_i^2}\right]^4$ If r is in $[R_o, R_i]$

$N_{T_i} = 0$ If r is not in $[R_o, R_i]$

Let $\hat{U}_S$ be the unit normal to the ellipsoid at the station



(7) $U_S = (ABCD)* \begin{pmatrix} s_1 \\ s_2 \\ s_{3/(1-e^2)} \end{pmatrix}$ s_i computed from (3) with $h_s=0$

(8) $k = |r_{s_o}|\sin Z_V = R_o \sin Z_V$

(9) $\cos Z_V = \hat{U}_S * \hat{\rho}$

(10) The refraction model is $(1+C_R)\,\Delta f_R$

$\Delta f_R = \dfrac{-f_s}{c}\,\Delta\dot{R}$ range rate

$\Delta f_R = \Delta R$ range

$\Delta f_R = \Delta R(t_n) - \Delta R(t_{n-1})$ range difference

ΔR and $\Delta\dot{R}$ are obtained by evaluating the integrals

(11) $$\Delta R = \int_{r_{station}}^{r_{satellite}} \frac{(n-1)\quad r^{*}dr}{(r^{*}r-k^2)^{1/2}}$$ (12) $$\dot{\Delta R} = k\dot{k}\int_{r_{station}}^{r_{satellite}} \frac{(n-1)\quad r^{*}dr}{(r^{*}r-k^2)^{3/2}}$$

where

$$k\dot{k} = \frac{-R_o^2}{|\rho|} \cos Z_V \,[(I-\hat{\rho}\,\hat{\rho}^{*})\dot{\rho} - \bar{\omega}\, x\rho]^{*}\hat{U}_S$$

Letting $R_o = r_{station}$ and $R_i = r_{satellite}$ we can compute range and range rate corrections by

(13) $$\Delta R = \int_{R_o}^{R_i} \frac{(n-1)\ r^{*}dr}{(r^{*}r-k^2)^{\frac{1}{2}}} = \sum_{i=1}^{2} \frac{N_i}{(R_o^2 - R_i^2)^4} \sum_{j=0}^{4} \frac{1}{2j+1} \binom{4}{j} (r^{*}r-k^2)^{j+\frac{1}{2}} (k^2-R_i^2)^{4-j} \Bigg|_{R_o}^{R_i}$$

(14) $$\frac{\Delta\dot{R}}{k\dot{k}} = \int_{R_o}^{R_i} \frac{(n-1)\ r^{*}dr}{(r^{*}r-k^2)^{3/2}} \quad \sum_{i=1}^{2} \frac{N_i}{(R_o^2 - R_i^2)^4} \sum_{j=0}^{4} \frac{1}{2j-1} \binom{4}{j} (r^{*}r-k^2)^{j-\frac{1}{2}} (k^2-R_i^2)^{4-j} \Bigg|_{R_o}^{R_i}$$

DATA TYPE FORMULATION

Letting D_9 denote range difference and D_7 denote range rate we have

(15) $$D9 = \left[|\rho| - \frac{c}{f_{oo}}\, f_b(t-TCA) - \frac{c}{2f_{oo}}\, \dot{f}_b(t-TCA)^2 + (1+c_R)\Delta f_R\right]_{t_{i-1}}^{t_i}$$

where f_{oo} is an input quantity usually taken to be E +(06) so that the bias solution will be in ppm.

Partial derivatives are given by

(16) $$\frac{\partial D9}{\partial q} = \frac{\partial[\,]_i}{\partial q} - \frac{\partial[\,]_{i-1}}{\partial q}$$ q = orbit set (p), station set (r_{so}), c_R, f_b or $\dot{f}_b$.

(17) $\dfrac{\partial[\,]}{\partial p} = \hat{\rho}^* \dfrac{\partial x}{\partial p}$

(18) $\dfrac{\partial[\,]}{\partial\, r_{s_o}} = -\hat{\rho}^* (ABCD)^*$

(19) $\dfrac{\partial[\,]}{\partial c_R} = \Delta f_R$

(20) $\dfrac{\partial[\,]}{\partial f_b} = \dfrac{-c}{f_{oo}} (t-TCA)$

(21) $\dfrac{\partial[\,]}{\partial \dot{f}_b} = \dfrac{-c}{2f_{oo}} (t-TCA)^2$

The vacuum received frequency using a transmitted frequency of f_s is given by

(22)
$$f_R = f_S \frac{\left(1 + \dfrac{\hat{\rho} * \dot{r}_s(t_r)}{c}\right)}{\left(1 + \dfrac{\hat{\rho} * \dot{r}(t_e)}{c}\right)}$$

where $\rho(t_r) = r(t_e) - r_s(t_r)$

$(t_e) = t_r - \dfrac{(\rho^*\rho)^{\frac{1}{2}}}{c}$ = emitted time

t_r = received time

computing $\dot{\rho}$ from the expression for ρ and taking into account the change in t_e as a function of t_r gives

(23)
$$\dot{\rho} = \frac{\dot{r}(t_e) - \dot{r}_s(t_r) + (\hat{\rho}^*/c)\, \dot{r}_s(t_r)\dot{r}(t_e) - (\hat{\rho}^*/c)\dot{r}(t_e)\dot{r}_s(t_r)}{\left(1 + \dfrac{\hat{\rho}^*\dot{r}(t_e)}{c}\right)}$$

$$1 - \frac{\hat{\rho}^*\dot{\rho}}{c} = \frac{1 + \dfrac{\hat{\rho}^*\dot{r}(t_e)}{c} - \left(\dfrac{(\hat{\rho}^*\dot{r}(t_e)}{c} - \dfrac{\hat{\rho}^*\dot{r}_s(t_e)}{c}\right)}{1 + \dfrac{\hat{\rho}^*\dot{r}(t_e)}{c}}$$

$$= \frac{1 + \hat{\rho}^* \dot{r}_s(t_r)/c}{1 + \hat{\rho}^* \dot{r}\ (t_e)/c}$$

$$= \frac{f_r}{f_s}$$

Thus if $\dot{\rho}$ is computed to first order in 1/c f_r/f_s will be given to second order in 1/c.

$$(24) \qquad \dot{\rho} \underset{\text{order}}{\overset{\text{1st}}{=}} \dot{r}(t_e) - \dot{r}_s(t_r) - (\hat{\rho}^*/c)\ (\dot{r}(t_e) - \dot{r}_s(t_r))\dot{r}(t_e)$$

Adding in the contribution of frequency bias and refraction gives the formulation for range rate as

$$(25) \qquad D_7 = f_s(1 + f_b/f_{oo} + \frac{\dot{f}_b}{f_{oo}}(t\text{-}TCA))\ (1 - \hat{\rho}^*\dot{\rho}/c) + (1 + c_R)\Delta f_R$$

This formula is accurate to second order in 1/c where as the corresponding formula (15) for range difference is accurate to all orders in 1/c.

$$(26) \qquad \frac{\partial D_7}{\partial p} = \frac{-f_s}{c}\left[\frac{\dot{\rho}^*}{|\rho|}(I - \hat{\rho}\hat{\rho}^*)\frac{\partial r}{\partial p} + \hat{\rho}^*\frac{\partial \dot{r}}{\partial p}\right]$$

$$(27) \qquad \frac{\partial D_7}{\partial r_{s_o}} = \frac{f_s}{c}\left[\frac{\dot{\rho}^*}{|\rho|}(I - \hat{\rho}\hat{\rho}^*) + \hat{\rho}^* x \bar{\omega}^*\right](ABCD)^*$$

$$(28) \qquad \frac{\partial D_7}{\partial c_R} = \Delta f_R$$

$$(29) \qquad \frac{\partial D_7}{\partial f_b} = \frac{f_s}{f_{oo}}\ (1 - \hat{\rho}^*\dot{\rho}/c)$$

$$(30) \qquad \frac{\partial D_7}{\partial \dot{f}_b} = \frac{f_s}{f_{oo}}\ (1 - \hat{\rho}^*\dot{\rho}/c)\ (t\text{-}TCA)$$

The primary output statistics from the filtering process are filtered noise and the RWS of residuals.

$$(31) \qquad RWS = \left[\sum_{i=1}^{N_{obs}} \omega_i^2\ (D_{obs} - D)_i^2/N_{obs}\right]^{1/2}$$

where D is computed on the reference ephemeris and ω_i is the computed weight.

$$\text{(32)} \quad \text{Filtered Noise} = \sum_{i=1}^{N_{obs}} \left[1/(N_{obs}\omega_i^2) \right]^{1/2}$$

= standard deviation for an observation from the pass.

As the RWS of adjusted residuals converges to one during the filtering process and the weights are approximately constant we have

$$\omega \left[\sum_{i=1}^{N_{obs}} (D_{obs} - D_{adjusted})_i^2 / N_{obs} \right]^{\frac{1}{2}} = 1$$

or RMS of adjusted residuals = $\frac{1}{\omega}$ = standard deviation
= Filtered Noise

SOLUTION OF NORMAL EQUATIONS

The solution area solves an equation containing from six to thirty nine dynamic parameters, several hundred bias parameters and several sets of station coordinates. To accomplish this the program performs bias and station parameter elimination in order to keep the matrix requiring inversion under dimension of forty. Letting "o" stand for the orbit (dynamic) parameters and "b" for bias we have the normal equation

$$\begin{bmatrix} B_{oo} & B_{ob} \\ B_{bo} & B_{bb} \end{bmatrix} \begin{bmatrix} \Delta p_o \\ \Delta p_b \end{bmatrix} = \begin{bmatrix} E_o \\ E_b \end{bmatrix}$$

The bias terms are eliminated from the above equation and the elimination equations are saved in order to recover bias solutions.

$$\text{(33)} \qquad \Delta p_b = B_{bb}^{-1} (E_b - B_{bo}\Delta p_o)$$

$$\text{(34)} \qquad (B_{oo} - B_{ob}B_{bb}^{-1}B_{bo})\,\Delta p_o = E_o - B_{ob}B_{bb}^{-1}E_b$$

$$\text{or } B_{oo}^{\text{Eliminated}} \quad \Delta p_o = E_o^{\text{Eliminated}}$$

Just prior to bias elimination pass matrices are expanded from their basic set of parameters to the desired solution set. This is a major task in the solution area. The solution set may consist of orbital elements at a different epoch, multiple drag parameters due to drag segmentation, multiple thrust, polar motion parameters, up to twenty gravity parameters, bias parameters and station coordinates.

Subsequent to bias elimination pass matrices are summed to form an arc normal matrix. Station coordinate sections of the matrix are summed over each station and eliminated just prior to matrix inversion. Station solutions are obtained by backsubstitution. Direct observattion of all parameters is permitted in the form of apriori sigma input for each parameter. The inverse square of the input sigma is added to the diagonal term of the normal matrix prior to inversion. This corresponds to an observation of the present value of a parameter with the input sigma as the standard error of observation.

NAVIGATION SOLUTIONS

The primary diagnostic output is a set of navigation solutions. A navigation solution is carried out for each pass of data and consists of determining the receiver motion, in the along track and radial directions, which minimizes residuals for the refined ephemeris. The receiver motion is interpreted as satellite ephemeris error as the receiver location is actually well known. The radial and along track directions used are the range and range rate vectors at TCA discussed earlier under filtering. The procedure is implemented by saving parts of the bias elimination equations (33)

$$B_{bb}, B_{bo}, E_b$$

and adjusting for the orbit solution Δ_{po} obtained at the time of arc matrix inversion.

$$B_{bb} \ \Delta b \approx E_b - B_{bo} \ \Delta po$$

The B_{bb} section contains earth fixed receiver coordinates which are transformed to the local TCA frame. Solutions are generated for radial, along track, frequency and refraction error. Diagnostic cards containing this information and filter statistics are generated at this time.

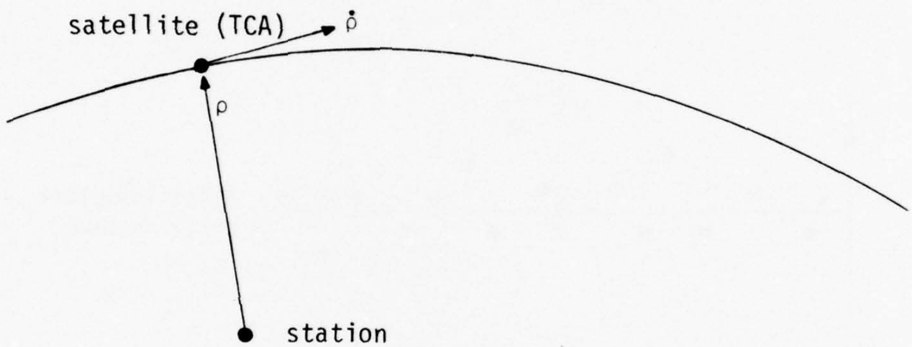
STATION DIAGNOSTIC CARDS

Station Analysis cards can be punched on option and contain diagnostic informaion useful to the satellite tracking stations. The card values are as follows:

1. STA - Station number extracted from the header message of the raw data.

2. Time (hr, min) - Time of the first data point.

3. TCA (sec) - Time of closest appraoch of the satellite obtained from the satellite orbit by searching for the point in time where the range (ρ) and range rate ($\dot{\rho}$) vectors are perpendicular to each other.



4. FREQ(mc) - The Q number from the raw data header message is used to determine a base frequency. The frequency given is this base frequency rounded to the nearest MHz.

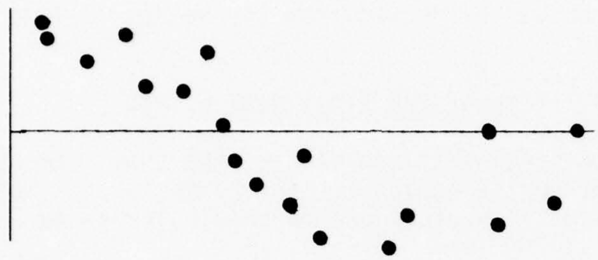
5. EL (deg) - The satellite elevation at TCA computed from the satellite orbit.

6. PTS. Good - Total number of points left after passing through the Celest point filtering process.

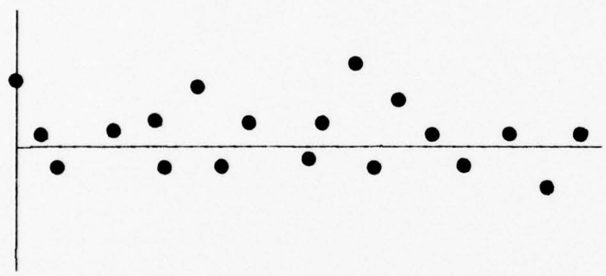
a. Points are filtered out in the Celest Pre-Processor if information is missing, values are to large or the data fails a monotone test.

b. Points are filtered out in the Celest Filter by removing orbit error from the residuals and testing against 2.0 sigma.

7. Filt. Noise (cps/MHz or m) - Filtered noise is the standard deviation on the data after modeling error has been removed. In the case of range rate this value is given in units of cps/MHz. In the cases of range difference the value is given in units of meters.



Before modeling error removed



After modeling error removed

Filtered Noise = RMS of residuals after modeling error is removed.
This value is scaled to 1 MHz for Doppler data.

8. ELT.(m) - This is the along track navigation error determined from the refined orbit. Holding the orbit fixed the station is allowed to move in the along track (ρ) and slant range ($\dot{\rho}$) directions, in order to best fit the data from the pass. Since this movement is from a known position the result is tabulated as along track (ELT) and slant range (ELR) errors. The values represent a measurement of how well the final refined orbit fits the data of a given pass. (see diagram for #3)

9. ELR.(m) - Slant range error.

10. DLT. F. (ppm)[1] - Delta frequency is the value of the frequency bias determined in the above navigation solution. Assuming that the satellite frequency has a constant bias during a given pass, then this number represents that bias in parts per million.

11. ACT - Action taken in the course of point filtering. Action label described below.

A - No TCA

B - Rejected in filter because to many points were filter out.

D - Rejected on TCA zenith angle test.

E - Pass not balanced, i.e. the difference between the number of points on one side of TCA and the number of points on the other is greater than the balanced pass tolerance.

Passes are rejected for reasons other than the above, in the Pre-Processor. These reasons are listed in Pre-Processor under Reject Codes but no indication is given on the Station Analysis cards.

INTEGRATION

For ephemeris computation a variable order routine is used on a 14 digit machine. Usually the order is set to ten and navigation type satellites use a 60 sec step size. The ephemeris can be computed to a one meter accuracy and perturbations of orbit constants to one part in 10^6.

The integration routine is a Gauss Jackson technique using backward differences and follows the basic pattern of first initializing a backward difference table to the order (N) of the process

(1) A nominal value of oscillator frequency offset (in parts per million) is associated with each satellite. On the basis of the Doppler data from a given pass, a correction, DLT. F. is calculated. The corrected absolute offset, in parts per million, for the pass is the algebraic sum of the nominal offset and DLT. F.
Absolute offset = Nominal offset + DLT. F.
= Δv_s + DLT. F.
The nominal offset is always negative. For example, if the nominal offset for a satellite is -80 ppm, a DLT. F. of +.04 would indicate an absolute offset of -80+.04 = -79.96 ppm.

then

1. Extrapolates the difference table from line n to n+1.

$$\nabla^{-1} \ddot{x}(n) = \nabla^{-1} \ddot{x}(n-1) + \nabla^{\circ} \ddot{x}(n)$$

$$\nabla^{-2} \ddot{x}(n) = \nabla^{-2} \ddot{x}(n-1) + \nabla^{-1} \ddot{x}(n)$$

$$\nabla^{N} \ddot{x}(n+1) = \nabla^{N} \ddot{x}(n)$$

$$\nabla^{k} \ddot{x}(n+1) = \nabla^{K} \ddot{x}(n) + \nabla^{K+1} \ddot{x}(n+1) \quad K=N-1,\ldots 0$$

2. Computes position and velocity by

$$X_{n+1} = h^2 \nabla^{-2} \ddot{x}_n + h^2 \sum_{K=0}^{N} c_K \nabla^K \ddot{x}_{n+1}$$

$$\dot{x}_{n+1} = h \nabla^{-1} \ddot{x}_n + h \sum_{K=0}^{N} a'_K \nabla^K \ddot{x}_{n+1}$$

where

$$a'_K = a_{K+1} \quad a'_0 = 1 + a_1 = 1/2 \quad a_0 = 1 \; a_1 = -1/2$$

$$c_K = \sum_{e=0}^{K+2} a_e a_{K+2-e}$$

$$a_K = - \sum_{j=1}^{K} \frac{a_{K-j}}{1+j}$$

3. Uses the force function, G, from $\ddot{x} = G(x,\dot{x},t)$ to compute $\ddot{x}(n+1)$ and determine the difference between the computed and extrapolated values of acceleration. The backward difference table is adjusted due to this difference.

4. The process described in 2 and 3 is continued until the desired number of iterations is reached. The final result from 2 is written on the trajectory file and the process terminates by carrying out step 3.

COORDINATE AND TIME SYSTEMS

The reference frame used for integration is an inertial frame defined by the mean equator and equinox of 1950.0 the sun and moon coordinates are in the 1950.0 system using Ephemeris Time (ET). The time system for Doppler observations is Universal Time Coordinates (UTC) and thus the integration time is UTC. The difference between UTC and ET is presently 46.15 sec and is not presently adjusted for when using the

and moon coordinates in the force model. The gravity force is computed by rotating the satellite inertial position to the earth fixed frame aligned with the Greenwich meridian and using the instantaneous earth spin axis. The difference between UTC and UT1 is taken into account when computing the rotation. The calculation of residuals is made by bringing station coordinates, referenced to the CIO pole, into the inertial system. The transformation between the earth fixed system using the conventional International Origin (CIO) and the inertial system is given by

$$X_{EF} = ABCDX_{\text{Inertial}} \tag{35}$$

where
D = general precession
C = nutation
B = rotation from true inertial equator and equinox of a given time to Greenwich at that time
A = polar motion to the CIO pole using polar motion values routinely solved for in Defense Mapping Agency Navigation Satellite processing.

FORCE EQUATION

The force equation is represented as

$$\ddot{x} = G(x,\dot{x},t) = A_e + A_s + A_m + A_d + A_r + A_{ts} + A_{tm} + A_t \tag{36}$$

where the accelerations are due to earth, sun and moon gravity, atmospheric drag, radiation pressure, solar and lunar tidal distortion, and vehicle thrust.

A_e (EARTH GRAVITY)

The earth's potential in an earth fixed frame is

$$V = \mu \sum_{n=0}^{N} \sum_{m=0}^{n} \left[\frac{a_e^n \, C_{nm} P_n^m \left(\frac{g'}{|r|}\right) \cos(m\lambda)}{|r|^{n+1}} + \frac{a_e^n \, S_{nm} P_n^m \left(\frac{g'}{|r|}\right) \sin(m\lambda)}{|r|^{n+1}} \right] \tag{37}$$

where $r = \begin{pmatrix} e' \\ f' \\ g' \end{pmatrix}$ (earth fixed coordinates)

a_e = Semi-major axis of the earth
= 6378.145 km
P_n^m = Legendre polynomial
μ = Earth's gravity constant
= 398601.0
λ = Longitude with respect to Greenwich
C_{nm}, S_{nm} = Gravity constants

Since the coordinate system has its center at the earth's center of gravity we have $C_{10}=C_{11}=S_{11}=0$. The earth's gravitational acceleration can now be given by

$$(38) \qquad A_e = \nabla_I V = \begin{pmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial V}{\partial z} \end{pmatrix}$$

where ∇_I is the inertial gradient. For the purpose of calculation we rewrite (37) as

$$(39) \qquad V = \sum_{n=0}^{N} \sum_{m=0}^{n} [C_{nm}U_n^m + S_{nm}V_n^m] \quad n \neq 1$$

noting the $V_n^{\circ}=C_{11}=C_{10}=S_{11}=0$.

Define the transformation E by

$$(40) \qquad E = \frac{BCD}{a_e}$$

Introduce the longitude by

$$(41) \qquad \begin{aligned} c(\lambda) &= \sin(\theta) \cos(\lambda) = \frac{a_e}{|r|} \sum_{i=1}^{3} E_{1i}\, x_i \\ s(\lambda) &= \sin(\theta) \sin(\lambda) = \frac{a_e}{|r|} \sum_{i=1}^{3} E_{2i}\, x_i \end{aligned}$$

where

$$\theta = \cos^{-1}\left(\frac{g'}{|r|}\right)$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \text{inertial components of } r .$$

Using the recurrence relations for Legendre polynomials we have recurrence relations for U and V given by

$$(42) \qquad \begin{aligned} U_{n+1}^m &= \frac{\rho}{(n-m+1)} \left[\frac{g'}{|r|} (2\,n+1)\, U_n^m - (n+m)\, \rho\, U_{n-1}^m\right] \\ V_{n+1}^m & \quad \frac{\rho}{(n-m+1)} \left[\frac{g'}{|r|} (2\,n+1)\, V_n^m - (n+m)\, \rho\, V_{n-1}^m\right] \end{aligned}$$

$$U_{n+1}^{n+1} = (2\ n+1)\ \rho[U_n^n\ C(\lambda) - V_n^n\ S(\lambda)]$$

(43)

$$V_{n+1}^{n+1} = (2\ n+1)\ \rho[V_n^n\ C(\lambda) - U_n^n\ S(\lambda)]$$

where $\rho = \frac{a_e}{|r|}$

Equation (42) is called Horizontal stepping and (43) is called Diagonal stepping.

Using the values

$$U_0^0 = \mu/|r| \qquad U_1^0 = \frac{\mu a_e g'}{|r|^3}$$

$$V_0^0 = 0 \qquad V_1^0 = 0 .$$

we start at n=1, m=0 in the horizontal stepping equation and compute U_i^0, V_i^0 for i=2,3...N. We then utilize diagonal stepping and calculate U_1^1, V_1^1. Returning to horizontal stepping enables the calculation of U_i^1, V_i^1 for i = 2,3,...N. This process is repeated until m = M, where $M \leq N$. Note that

$$U_n^{-m} = \frac{(-1)^m (n-m)!}{(n+m)!} \quad U_n^m$$

$$V_n^{-m} = \frac{(-1)^{m+1} (n-m)!}{(n+m)!} \quad V_n^m$$

We can now compute (38) by

$$\text{(44)} \qquad \nabla_I V = E^* \sum_{n=0}^{N} \sum_{m=0}^{n} [C_{nm} a_e \nabla U_n^m + S_{nm} a_e \nabla V_n^m]$$

where ∇ is the earth fixed gradient.

The recurrence relations for U and V can be given by

(45)

$$a_e \nabla U_n^m \begin{bmatrix} \frac{1}{2} A_n^m U_{n+1}^{m-1} & -\frac{1}{2} U_{n+1}^{m+1} \\ -\frac{1}{2} A_n^m V_{n+1}^{m-1} & -\frac{1}{2} V_{n+1}^{m+1} \\ & -(n-m+1) U_{n+1}^m \end{bmatrix}$$

$$a_e \nabla V_n^m \begin{bmatrix} \frac{1}{2} A_n^m V_{n+1}^{m-1} & -\frac{1}{2} V_{n+1}^{m+1} \\ \frac{1}{2} A_n^m U_{n+1}^{m-1} & +\frac{1}{2} U_{n+1}^{m+1} \\ & -(n-m+1) V_{n+1}^m \end{bmatrix}$$

where $A_n^m = (n-m+1)\ (n-m+2)$

A_s, A_m (SUN AND MOON GRAVITY)

Coordinates of the sun and moon are stored on the Sun-Moon file at one day and one half day intervals respectively. A sixth order Lagrangian interpolation procedure is used to obtain the values at any time. The sun's acceleration on the satellite is given by

(46) $$A_s = -\mu_s \left(\frac{r-r_s}{|r-r_s|^3} + \frac{r_s}{|r_s|^3} \right), \quad \mu_s = .1330614\ E(12)$$

The expression for the moon is similar with $\mu_m = .490074\ E(04)$.

A_{ts}, A_{tm} (TIDAL DISTORTION)

The gravitational (tidal) attraction of the sun and moon causes the earth to become elongated on an axis pointing toward the disturbing body. The redistribution of mass results in a perturbation of the earth's own gravitational field, which is represented by the potentials

(47) $$U_s = -k_L \frac{\mu_s}{|r_s|^3} \frac{a_e^5}{|r|^3} P_2(\hat{r}*\hat{r}_s) \quad U_m = -k_L \frac{\mu_m}{|r_m|^3} \frac{a_e^5}{|r|^3} P_2(\hat{r}*\hat{r}_m)$$

where k_L is Love's constant and P_2 is the Legendre polynomial.

The associated acceleration on the satellite is given by

$$(48) \quad A_{ts} = k_L \frac{\mu_s}{|r_s|^3} \frac{a_e^5}{|r|^5} [(-\frac{15}{2}(\hat{r}*\hat{r}_s)^2+3/2)r+3(r*\hat{r}_s)\hat{r}_s]$$

with a similar expression for A_{tm}.

A_r (RADIATION PRESSURE)

A shadow test is performed to determine if the satellite is in sunlight or shadow. If $r\cdot r_s \geq 0$ then the satellite is in sunlight. If $r\cdot r_s < 0$ then we compute $|rx\hat{r}_s|$. If $|rx\hat{r}_s|$ is less than a_e the satellite is in shadow and if not then it is in sunlight. When the satellite is in sunlight we compute the radiation pressure acceleration by

$$(49) \qquad A_r = k_r \frac{s}{m} \; 10^{14} \frac{(r-r_s)}{|r-r_s|^3}$$

where s is the satellite cross-sectional area and m is satellite mass.

A_r is set to zero in the shadow.

A_d (ATMOSPHERIC DRAG)

The relative velocity of the satellite with respect to the atmosphere is

$$V_r = \dot{r} - \varpi x r$$

where $(r,\dot{r})$ is the inertial satellite position, velocity and ϖ is $(CD)*\begin{pmatrix}0\\0\\ \tilde{\omega}\end{pmatrix}$. The acceleration of the satellite due to drag is

$$(50) \qquad A_d = -C_D \rho \frac{s}{2m} |V_r| \; V_r$$

where m is the satellite mass, s is the cross-sectional area and ρ the atmospheric density function defined by

$$\rho = \mathrm{Exp}\,[Ah-B-(Ch^2+Dh-E)^{\frac{1}{2}}]$$

The height h is the satellites geocentric altitude above its subpoint given by

$$h = |r| \left[1 - \frac{a_e}{\left(|r|^2 + \frac{e^2}{1-e^2} Z^2 \right)^{\frac{1}{2}}} \right]$$

$$= |r| \quad -R$$

where R is the distance from the earth's center to the satellites sub-point.

$$R = \frac{a_e}{\left(1 + \frac{e^2}{1-e^2} \sin^2 \theta \right)^{\frac{1}{2}}}$$

θ = geocentric latitude

A_t (VEHICLE THRUST)

(51) $\quad A_t = RA$

where

$$R = [\hat{r}, \hat{\dot{r}}, \widehat{r x \dot{r}}]$$

$$A = \begin{pmatrix} A_r \\ A_{\dot{r}} \\ A_{rx\dot{r}} \end{pmatrix}$$

The values for A are given by input as constants in the radial, tangential and out of plane directions.

VARIATIONAL EQUATIONS

Partial derivatives are computed by the same integration procedure simultaneously with the orbit integration. The variational equations are

(52) $$\ddot{h} = \frac{\partial G}{\partial x} h + \frac{G}{\partial \dot{x}} \dot{h} + \frac{G}{\partial p}$$

where

$$\frac{\partial G}{\partial p} = \begin{cases} 0 & p = \text{osculating orbital element} \\ \dfrac{\partial \text{ Model}}{\partial \text{ Model Parameter}} & p = C_D,\ A \text{ or } k_r \end{cases}$$

EARTH GRAVITY VARIATION

$$(53)\qquad \frac{\partial A_e}{\partial x} = \nabla_I^2 V = E^\star \sum_{n=0}^{N} \sum_{m=0}^{n} [C_{nm} a_e^2 \nabla^2 U_n^m + S_{nm} a_e^2 \nabla^2 V_n^m] E$$

where

$$a_e^2 \nabla^2 U_n^m = \begin{bmatrix} \frac{1}{2} A_n^m a_e \nabla U_{n+1}^{m-1} & -\frac{1}{2} a_e \nabla U_{n+1}^{m+1} \\ -\frac{1}{2} A_n^m a_e \nabla V_{n+1}^{m-1} & -\frac{1}{2} a_e \nabla V_{n+1}^{m+1} \\ \multicolumn{2}{c}{-(n-m+1) a_e \nabla U_{n+1}^m} \end{bmatrix}$$

(54)

$$a_e^2 \nabla^2 V_n^m = \begin{bmatrix} \frac{1}{2} A_n^m a_e \nabla V_{n+1}^{m-1} & -\frac{1}{2} a_e \nabla V_{n+1}^{m+1} \\ +\frac{1}{2} A_n^m a_e \nabla U_{n+1}^{m-1} & +\frac{1}{2} a_e \nabla U_{n+1}^{m+1} \\ \multicolumn{2}{c}{-(n-m+1) a_e \nabla V_{n+1}^m} \end{bmatrix}$$

The partials for C_{nm} and S_{nm} are

$$\frac{\partial \nabla_I V}{\partial C_{nm}} = E^\star a_e \nabla U_n^m \qquad (n,m) \neq (0,0)$$

(55)

$$\frac{\partial \nabla_I V}{\partial S_{nm}} = E^\star a_e \nabla V_n^m$$

$$\frac{\partial \nabla_I V}{\partial \mu} = \nabla_I V / \mu$$

SUN AND MOON GRAVITY VARIATIONS

Setting $p_s = |r - r_s|$ the derivatives are

$$\nabla A_{s,x} = \frac{-\mu_s}{p_s^5}\begin{bmatrix} -3(x-x_s)^2 + p_s^2 \\ -3(x-x_s)\ (y-y_s) \\ -3(x-x_s)\ (z-z_s) \end{bmatrix}$$

(56) $$\nabla A_{s,y} = \frac{-\mu_s}{p_s^5}\begin{bmatrix} -3(x-x_s)\ (y-y_s) \\ -3(y-y_s)^2 + p_s^2 \\ -3(y-y_s)\ (z-z_s) \end{bmatrix}$$

$$\nabla A_{s,z} = \frac{-\mu_s}{p_s^5}\begin{bmatrix} -3(x-x_s)\ (z-z_s) \\ -3(y-y_s)\ (z-z_s) \\ -3(z-z_s)^2 + p_s^2 \end{bmatrix}$$

with similar results for the moon.

SUN AND MOON TIDAL VARIATIONS

(57) $$\nabla A_{ts} = \frac{k_L}{2}\frac{\mu_s}{|r_s|^3}\frac{a_e^5}{|r|^5}\Big\{[3-15(\hat{r}^*\hat{r}_s)^2]\ I + 6\ \hat{r}_s\ \hat{r}_s^* + [105(\hat{r}^*\hat{r}_s)-15]\ \hat{r}\ \hat{r}^* - 30\ \hat{r}^*\hat{r}_s\ [\hat{r}\ \hat{r}_s^* + \hat{r}_s\ \hat{r}^*]\Big\}$$

with similar results for the moon.

RADIATION PRESSURE VARIATIONS

Setting $p_s = |r - r_s|$ the derivatives are

$$\nabla A_{r,x} = \frac{k_r S\ 10^{14}}{m\ p_s^5} \begin{pmatrix} -3(x-x_s)^2 + p_s^2 \\ -3(x-x_s)\ (y-y_s) \\ -3(x-x_s)\ (z-z_s) \end{pmatrix}$$

(58)
$$\nabla A_{r,y} = \frac{k_r S\ 10^{14}}{m\ p_s^5} \begin{pmatrix} -3(y-y_s)\ (x-x_s) \\ -3(y-y_s)^2 + p_s^2 \\ -3(y-y_s)\ (z-z_s) \end{pmatrix}$$

$$\nabla A_{r,z} = \frac{k_r S\ 10^{14}}{m\ p_s^5} \begin{pmatrix} -3(z-z_s)\ (x-x_s) \\ -3(z-z_s)\ (y-y_s) \\ -3(z-z_s)^2 + p_s^2 \end{pmatrix}$$

$$\frac{\partial A_r}{\partial k_r} = \frac{S\ 10^{14}}{m} \frac{(r-r_s)}{p_s^3}$$

DRAG VARIATIONS

$$\frac{\partial A_d}{\partial (r\dot{r})} = \frac{-C_D S}{2m} \left[\rho |V_r| \frac{\partial V_r}{\partial (r\dot{r})} + \rho V_r \frac{\partial |V_r|}{\partial (r\dot{r})} + |V_r| V_r \frac{\partial \rho}{\partial (r\dot{r})} \right]$$

(59)
$$\frac{\partial A_d}{\partial C_D} = -\frac{\rho S}{2m} |V_r| V_r$$

Since $V_r = \dot{r} - \tilde{\omega} x r = \dot{r} - \Omega r$

$$\Omega = (CD)^* \begin{bmatrix} 0 & -\tilde{\omega} & 0 \\ \tilde{\omega} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} (CD)$$

$$\frac{\partial V_r}{\partial r} = -\Omega$$

$$\frac{\partial V_r}{\partial \dot{r}} = I$$

$$\frac{\partial V_r}{\partial (r\dot{r})} = [-\Omega, I]$$

$$\frac{\partial |V_r|}{\partial (r\dot{r})} = \hat{V}_r^* \frac{\partial V_r}{\partial (r\dot{r})}$$

$$\frac{\partial \rho}{\partial (r\dot{r})} = \left[\frac{\partial \rho}{\partial r} \quad , \; 0\right]$$

$$\frac{\partial \rho}{\partial r} = \frac{\partial \rho}{\partial h} \; \frac{\partial h}{\partial r}$$

$$\frac{\partial \rho}{\partial h} = \rho[A-(ch + D/2)\;(ch^2+Dh-E)^{-\frac{1}{2}}]$$

$$\frac{\partial h}{\partial r} = \frac{1}{|r|^2}\,[h + \frac{(|r| - h)^3}{a_e^2}\,]r^* + \frac{(|r|- h)^3\; e^2\;(0,0,Z)}{|r|^2\; a_e^2\;(1-e^2)}$$

THRUST VARIATIONS

(60) $$\frac{\partial A_t}{\partial (r\dot{r})} = \frac{\partial R}{\partial (r\dot{r})}\; A$$

$$\frac{\partial A_t}{\partial A} = R$$

Setting $r = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\frac{\partial R}{\partial (r\dot{r})} = \left[\frac{\partial \hat{r}}{\partial (r\dot{r})}, \frac{\partial \hat{\dot{r}}}{\partial (r\dot{r})}, \frac{\partial r\hat{x}\hat{\dot{r}}}{\partial (r\dot{r})}\right]$$

$$\frac{\partial \hat{r}}{\partial (r\dot{r})} = \left[\frac{\partial \hat{r}}{\partial r} \quad , \quad \frac{\partial \hat{r}}{\partial \dot{r}}\right] = \left[\frac{\partial \hat{r}}{\partial r} \; , \; 0\right]$$

$$\frac{\partial \hat{r}}{\partial r} = \frac{1}{|r|}\;[I - \hat{r}\;\hat{r}^*]$$

$$\frac{\partial \hat{\dot{r}}}{\partial (r\dot{r})} = [0, \frac{\partial \hat{\dot{r}}}{\partial \dot{r}}]$$

$$\frac{\partial \hat{\dot{r}}}{\partial \dot{r}} = \frac{1}{|\dot{r}|}\left[I-\hat{\dot{r}}\hat{\dot{r}}^{*}\right]$$

$$\frac{\partial \widehat{rx\dot{r}}}{\partial (r\dot{r})} = \left[\frac{\partial \widehat{rx\dot{r}}}{\partial r}, \frac{\partial \widehat{rx\dot{r}}}{\partial \dot{r}}\right]$$

$$\frac{\partial \widehat{rx\dot{r}}}{\partial r} = \frac{-\Omega_{\dot{r}}}{|rx\dot{r}|}\left[I - \frac{rr^{*}}{|rx\dot{r}|^{2}}\,\Omega^{*}_{\dot{r}}\,\Omega_{\dot{r}}\right]$$

$$\frac{\partial \widehat{rx\dot{r}}}{\partial \dot{r}} = \frac{\Omega_{r}}{|rx\dot{r}|}\left[I - \frac{\dot{r}\,\dot{r}^{*}}{|rx\dot{r}|^{2}}\,\Omega^{*}_{r}\,\Omega_{r}\right]$$

where $rx\dot{r} = \Omega_r\dot{r} = -\dot{r}xr = -\Omega_{\dot{r}}r$

$$\Omega_r = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

$$\Omega^{*}_{r}\;\Omega_{r} = \begin{bmatrix} x_2^2 + x_3^2 & -x_1x_2 & -x_1x_3 \\ -x_1x_2 & x_1^2 + x_3^2 & -x_2x_3 \\ -x_1x_3 & -x_2x_3 & x_1^2 + x_2^2 \end{bmatrix}$$

POLAR MOTION

Letting r_{s_0} denote the earth fixed station position and ABCD the inertial to earth fixed coordinate transformation

$$r_s = (ABCD)^{*}r_{s_0} \qquad r_{s_0} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

$$= (BCD)^{*}A^{*}r_{s_0}$$

$$A = \begin{bmatrix} 1 & -\omega_3 & \omega_2 \\ \omega_3 & 1 & -\omega_1 \\ -\omega_2 & \omega_1 & 1 \end{bmatrix} \qquad \omega_1 = \Delta q \quad \omega_2 = \Delta p$$

$$\omega_3 = \tilde{\omega} \quad (\Delta t + t\dot{\Delta} t)$$

t = time in sec. from the beginning of the year.

$$\frac{\partial r_s}{\partial p} = (BCD)^* \begin{bmatrix} 0 & -s_3 & \tilde{\omega} s_2 & \tilde{\omega} t s_2 \\ s_3 & 0 & -\tilde{\omega} s_1 & -\tilde{\omega} t s_1 \\ -s_1 & s_1 & 0 & 0 \end{bmatrix} = (BCD)^* Q$$

where $p = (\Delta q, \Delta p, \Delta t, \dot{\Delta} t)$ is the polar motion set.

$$\frac{\partial D_t}{\partial p} = \frac{\partial D_t}{\partial r_s} \frac{\partial r_s}{\partial p}$$

$$= \frac{\partial D_t}{\partial r_s} (BCD)^* A^* AQ$$

$$= \frac{\partial D_t}{\partial r_s} \frac{\partial r_s}{\partial r_{s_o}} AQ$$

$$\text{(61)} \qquad \frac{\partial D_t}{\partial p} = \frac{\partial D_t}{\partial r_{s_o}} AQ$$

Thus

$$B_{pp}(t) = \frac{\partial D_t^*}{\partial p} \frac{\partial D_t}{\partial p} = Q^*(t)\, A^*(t) \frac{\partial D_t^*}{\partial r_{s_o}} \frac{\partial D_t}{\partial r_{s_o}} A(t)\, Q(t)$$

$$= Q^*(t) A^*(t) B_{ss}(t)\, A(t)\, Q(t)$$

Evaluating AQ at TCA of a pass gives

$$\text{(62)} \qquad B_{pp} = \sum_{pass} B_{pp}(t) = Q^*(TCA)\; A^*(TCA)\; B_{ss}\; A(TCA)\; Q(TCA)$$

Equation (62) and other similar relations are now used to convert the station coordinate section of the pass normal equations into a polar motion section. This technique is used to incorporate polar motion parameters into Celest.

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